

# Game Theory and Applications

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### Preface

#### On the Dynamic Consistency of Optimal Monetary Policy ..... 1–15

*R. Cellini and L. Lambertini*

#### Abstract

The literature on the time inconsistency of optimal monetary policy puts forward the idea that a central bank may strategically exploit the first mover advantage against the private sector, manipulating expectations so as to achieve a higher level of employment and output. This view is questionable. The goal of the paper is that once one takes into account that expectations cannot be considered as choice variables, and the rule of expectation formation is common knowledge to all players, there may no longer be room for time inconsistency of monetary policy. The paper shows that the dynamic version of the basic model used in this literature is an optimal control model yielding a time consistent and stable solution to the central banker's problem, where prices are stable and the output reaches the full employment level in steady state. Then this idea is extended including a strategic private sector, which transforms the initial setup into a differential game. The open-loop Nash equilibrium is found, and it is proved that this game has a time consistent Stackelberg open-loop equilibrium with the bank leading, where, however, the bank cannot gain as compared to the simultaneous game. In the Stackelberg game in which the private sector leads, inflation arises in equilibrium, and output is below the full employment level. It is shown that in our setup optimal monetary policy is indeed to be time consistent, although it does not necessarily lead to the full stabilisation of prices. This holds irrespective of whether the game is played simultaneously or sequentially.

#### Uniform (0, 1) Two-person Poker Models ..... 17–37

*C. Ferguson, T. Ferguson and C. Gawargy*

#### Abstract

The study of two-person zero-sum poker models with independent uniform (0,1) hands goes back to Borel and von Neumann. E. Borel discusses a model of poker in Chapter 5, "Le jeu de poker", of his book *Applications aux Jeux de Hasard* (1938). Von Neumann presents his analysis of a similar model of poker in the seminal book on game theory by J. von Neumann and O. Morgenstern, *Theory of games and economic behavior* (1944). Most subsequent work on these models has been to extend the Borel model to allow several rounds of betting or more bet sizes. The von Neumann model, though more closely tied to actual play, is harder to treat mathematically. In this paper we solve several extensions to the von Neumann model. See Ferguson and Ferguson (2003) for a discussion and comparison of these two models.

<b>A Tax Game in a Cournot Duopoly</b> .....	<b>39–47</b>
<i>A. Galegov, A. Garnaev</i>	

### Abstract

In many countries the tax rate depends on the tax base. In Russia in 2003, so called simplified tax system, which consists of two tax rates (6 and 15 percents), was introduced in order to support small business. That is why some firms face up a problem of choosing one of them: either the pure profit tax rate (when firm pays tax from total revenue minus total cost) or the total revenue tax rate (when firm pays tax from total revenue). The total revue tax rate (6 percent) is less than the pure profit tax rate (15 percent), because the tax base in the first case is greater.

In the environment of competition, when there are few firms producing a homogeneous good for the market, the problem of choice becomes a game-theoretical problem since every firm should take into consideration behavior of its rival. We will call this situation a tax game. The aim of this paper is to generalize the problem of choice of the tax rate as well as the criteria of this choice for the competition case. In this paper we consider two plots of this game in the Cournot settings. The first one is a two stage game. On first stage firms plan their production in the Cournot settings for each combination of possible tax rates, while on the second stage they decide what tax rate will be better to use. The second plot is the one stage game where the firms choose the tax rate by optimal way after setting production plan.

<b>On a Corruption Game</b> .....	<b>49–58</b>
<i>A. Garnaev, A. Toricin</i>	

### Abstract

This paper deals with the analytical framework outlined by J. Buchanan and Y.J. Yoon (2000) to explore the symmetric "tragedies of the commons and the anti-commons" which was applied by A. Canavese (2004) to study the relationship between institutions and corruption. The aim of the paper is to generalize the Canavese model for the n corrupt agents case and for the case where there is a leader between the corrupted agents. We consider the effects of the corruption punishment on the resource allocation. Two ways of corruption punishment to discourage corrupt agents are studied. It is shown that corruption punishment based on the earnings collected from the bribes produces better allocation of resources than sanctions based on the number of corrupt acts committed. Also it is shown that if the corruption system has a hierarchical structure it can be easy destructed applying less facilities compare to less structured corruption system.

<b>The Quiet Accumulation Game on a Linear Graph a Special Case</b>	<b>59–77</b>
<i>K. Kikuta, W. Ruckle</i>	

### Abstract

A quiet accumulation game on a linear graph is formulated as a two person zero-sum game. Upper and lower bounds for the value of the game are given as well as pure strategies which assure those bounds by mixing them. Numerical examples are solved when the number of nodes is small.

<b>Monopolistic Payoffs in Cournot Oligopolies as a Result of Misspecified Demand .....</b>	<b>79–90</b>
<i>J.B. Krawczyk , F. Szidarovszky</i>	

#### **Abstract**

We study dynamic oligopolies under imperfect knowledge of the demand function. We show that if firms know the shape of the demand function but misspecify its scale they can enjoy stable monopolistic payoffs, which can reach the monopolistic levels. This may be exploited by firms with perfect knowledge of demand to drive the underinformed competitor's output down and to reap the benefits of producing more than under the Nash-Cournot equilibrium. Independently, this can also explain why some apparently competitive economies may produce at monopolistic levels.

<b>Game and Control Problems with Discontinuous Switches of Constraints for Control .....</b>	<b>91–122</b>
<i>S.S. Kumkov, A.N. Zharinov</i>	

#### **Abstract**

A nonlinear zero-sum differential game is analyzed. The problem is to construct maximal stable bridge  $W(v, M)$ , that is, a set maximal by inclusion, where from the first player guarantees reaching the terminal set  $M$  at the instant  $t$  despite of actions of the second player. The paper suggests a numerical algorithm for constructing attainability sets for nonlinear control and game problems in the plane. The control constraint depends on the phase vector, and this dependence can be discontinuous. Results of computation of some models are given and supported by several illustrative examples. Formalization of the control problem and the theoretic proofs of its reasonability are given. Possibilities and difficulties in justification of the algorithm for differential games are discussed.

<b>Numerical Study of the "Homicidal Chauffeur" Differential Game with the Reinforced Pursuer .....</b>	<b>123–152</b>
<i>V.S. Patsko, V.L. Turova</i>	

#### **Abstract**

One of well-known problems in the theory of differential games is a "homicidal chauffeur" problem proposed by R. Isaacs. The pursuer  $P$  (a car) and the evader  $E$  (a pedestrian) are moving in the plane. The linear velocity of the car is constant. The angular velocity of rotation of the linear velocity vector is bounded which means that the radius of turn of the car is bounded from below. The pedestrian is a non-linear object that can change the value and direction of his velocity instantaneously. The maximal possible value of the velocity is specified. By a given circular neighborhood of his current geometric position, player  $P$  tries to capture player  $E$  as soon as possible. The paper is devoted to the investigation of a time-optimal differential game in which the pursuer possesses increased control capabilities comparing to the classical homicidal chauffeur problem. Namely, the pursuer steers not only the angular velocity of turn but can additionally change the magnitude of his linear velocity. For such a new variant of the dynamics with non-scalar control of the

pursuer, a complete description of families of semipermeable curves is given and the dependence of the structure of level sets of the value function on a parameter that defines the bound on the magnitude of the pursuers velocity is explored by numerical methods.

## **Equilibrium in N-Players Competitive Silent Games of Timing 153–166**

*M. Sakaguchi, V.V. Mazalov*

### **Abstract**

The paper deals with the shooting contest for  $n$  players, competitive prediction of a random variable and war-games of attrition. The Nash equilibrium is derived among mixed strategies as a solution of the differential equation.

## **A Strategic Analysis of Product and Process Innovation with Spillovers ..... 167–177**

*L. Shravan*

### **Abstract**

We develop a two-stage non-cooperative R&D game of process and product innovation in a duopoly model which is distinct from Yin and Zuscovitch (1998) in the following way. Unlike the latter, we allow for process spillovers from which only the follower benefits in the model so that the follower's marginal cost of production is reduced not only by its own process innovation but also by a fraction of the leader's process investment. At the first stage of the game, the duopolists (the leader and the follower) will engage in product and process R&D. While product R&D is stochastic (in the sense that it realizes with a probability) and leads to the instantaneous discovery of a new product which leads to an outward shift of the firm's demand schedule, process R&D reduces the marginal cost of production with certainty. The two firms compete in the product market in the second stage. As in Yin and Zuscovitch (1998), results are derived by assuming that in the first stage the firm chooses product innovation taking process innovation as given and vice versa and finally the impact of spillovers on product and process strategies is found. Our results show that the spillover rate plays a critical role in analyzing the interplay between process and product innovations. The central contribution of our work is to offer a conceptual model for determining the impact of spillovers on the industry's innovation level and also for understanding the factors which might cause a firm to change its strategy from process to product when the spillover rate becomes small. Our results demonstrate that there exists a negative relationship between the spillover parameter and the product innovations of both the leader and follower. This suggests that we may observe switching behavior in an industry when the spillover rate becomes small.

## Values for TU Games with Linear Cooperation Structures .... 179–191

*E. Yanovskaya*

### Abstract

Aumann and Dreze (1975) were the first to introduce TU games with coalitional structures, into the study of value. They characterized the value assigning to each TU game with a coalitional structure the payoff vector consisting of the Shapley values (1953) of subgames defined by coalitions of the partition. Owen (1977) extended the Shapley value to games with a coalitional structure by a restriction of symmetry. Winter (1989) extended this approach to the games with an hierarchy of a finite sequence of partitions. In this paper we extend the Owen and the Winter values to a class of TU games with linear cooperation structures. A linear cooperation structure given exogenously means that if some players can cooperate with two non-intersecting coalitions then the coalitions cannot cooperate behind the players' backs. The symmetry across coalitions axiom used in Owen/Winter's characterizations of their values is not well defined for cooperation structures with intersecting coalitions. Thus, for such structures we add a new axiom stating independence of the values of coalitions whose players cannot be placed consecutively w.r.t. the linear order of players in which the coalitions of a given structure have no "holes". In the paper an axiomatization of the value for TU games with linear cooperation structures is given. A comparison of this value with the Myerson value for the class of connected cooperation graphs being simple chains is discussed.

## Oligopoly Competition under Environmental Design ..... 193–206

*N.A. Zenkevich, A.V. Zyatchin*

### Abstract

The present research deals with a problem of environmental design by means of a stochastic differential game. Firms competing a la Cournot over time produce their outputs which pollute the air. The paper studies two cases: perfect and imperfect information. In the first case, solving the Hamilton-Jacobi-Bellman equation, the Nash equilibrium in the stochastic differential game is obtained in an explicit form. In case of imperfect information, similar technique is applied, which provides the Bayesian equilibrium. Illustrative examples are presented.

## A Continuous Model of N-Person Prisoners' Dilemma ..... 207–242

*J. Zhao, M.N. Szilagyi and F. Szidarovszky*

### Abstract

The model in this paper can be considered as the extended version of the classical N-person Prisoners' Dilemma: a continuous N-person Prisoners' Dilemma. In the model each agent is assumed to be a mixture of six major personality types, and their state space is continuous. If continuous time scales are assumed, then the dynamism of the society can be modeled by a system of nonlinear ordinary differential equations. In the case of discrete time scales, our model is formulated as a system of difference equations. In certain special cases, our continuous model is equivalent to the discrete state models discussed earlier in the literature; therefore the results are the same. In general, the continuous state models generate different answers

than the corresponding discrete state models. Our model could be deterministic or stochastic, however the discrete model is always stochastic. The update of state in the continuous model depends on the agent's previous state, and the new state can be any value between 0 and 1. In contrast, the update of state in the discrete model can be only change from 0 to 1, from 1 to 0, or remain unchanged. An agent based simulation process is developed and applied to describe the dynamic changes in the structure of the society, as well as to analyze the long-term behavior of the agents.